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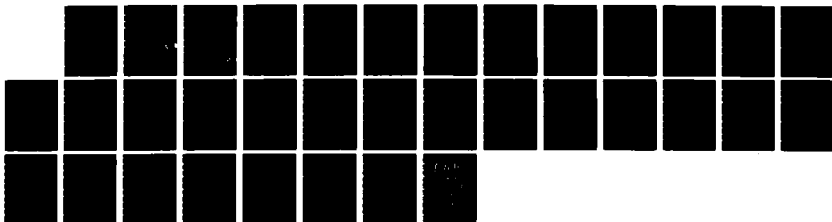
A MULTI USER RANDOM ACCESS COMMUNICATION SYSTEM FOR
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DEPT OF ELECTRICAL ENGINEERING D KAZAKOS ET AL
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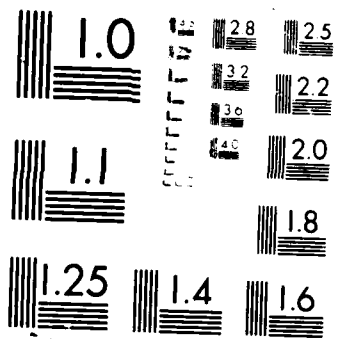
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A multi user random access communication system with a population of two classes of users is considered. It is assumed that packets generated by users from different classes have different priorities. Fast moving users in a mobile communication system, or high priority users in a static environment, might be members of the high priority class.

A binary feedback collision resolution algorithm is developed and both throughput and delay analysis are performed. Analytical results show that for the operation region of practical interest, the high priority class experiences significantly shorter delays, compared to the low priority one which maintains good delay characteristics.

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**A MULTI USER RANDOM ACCESS COMMUNICATION SYSTEM
FOR USERS WITH DIFFERENT PRIORITIES**

Submitted to:

Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, D.C. 20332

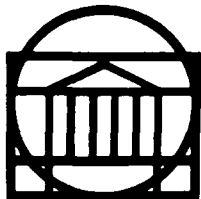
Attention: Major Brian W. Woodruff
USAF/NM

Submitted by:

D. Kazakos
Professor

I. Stavrakakis
Graduate Research Assistant

Report No. UVA/525656/EE87/102
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**SCHOOL OF ENGINEERING AND
APPLIED SCIENCE**

DEPARTMENT OF ELECTRICAL ENGINEERING

**UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901**



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I. Introduction

So far the existing literature on the multi user random access communication systems has been dealing with a homogeneous population of users [1]-[5]. There are many practical applications, however, where it is desired that some packets experience shorter delays than the average regular packet of the system. If all users are to use the same communication system, then the need for dividing the population of users into two classes arises.

There are cases of communication systems with homogeneous population of users where, at specific known time periods, the input traffic to the channel decreases significantly with respect to the nominal point of operation of the system. As a result, the average packet delay decreases but the utilization of the system decreases as well. Under those conditions, we can improve the utilization of the system by letting a second class of same priority users have access to the system. By controlling the rate of the input traffic coming from the second class, we can achieve induced average packet delays for both classes around the nominal point of the original class. In that case, the same algorithm applies to both classes and in fact we have a homogeneous user population. A second option is to adopt an algorithm that gives priority to the packets of the original class. In that case, it is expected that if the induced average packet delay of the original (high priority) class is around its nominal value, then the induced delays of the second class will be significantly larger. On the other hand, the low priority packet traffic, that induces the nominal average packet delay for the high priority class, is expected to be much larger than in the previous case of the equivalent classes. If the users of the second class can wait for the occurrence of the low traffic time periods of the original system, then it is reasonable to assume that those users can tolerate an additional delay of a small number of packet lengths. Thus, by using a system with users with different priorities, we can greatly increase the utilization of a system at essentially no cost.

In a mobile user environment where users move in and out of the range of the system, or move from region to region, fast moving users may need to experience shorter delays than the regular ones; this may be necessary to make packet transmission possible while the user is still inside the region. Also, users

that are close to the boundaries of a region and are going to move outside it, should experience shorter delays.

In a static user environment there are also cases in which some packets have high priority and should reach their destination faster than the regular ones. High priority packets can be those which are generated by high priority users (e.g. important users, or users that can pay more for better service), or can be packets that are generated by any user of the system but the information that is carried is characterized as important and deserves high priority in its transmission.

An important measure of performance of a communication system is the induced average packet delay. In some environments, there may exist strict constraints on the delay that some packets can tolerate. If a threshold is exceeded, the packet is considered to be lost and the average number of those packets can be a measure of performance. By considering that those special packets form a separate class which is given priority by the system, we might be able to reduce the induced delays of those packets below the rejection threshold and thus greatly improve the performance of the system.

In the next two sections the communication system and the suggested algorithm are described. In sections IV and V throughput and delay analysis are performed, while in the last section the results of the analysis are shown and conclusions are drawn.

II. The Communication System

We consider a large population of users that use a single communication channel. We assume that users which for some reason need to have some priority over the rest of the population, form the high priority class. It is assumed that the packet traffic generated by that class represents only a small percentage of the total traffic that is served by the system. In other words, we assume that the packets that need special service are rare and this is a realistic assumption at least for the environments that were described above.

The input traffic to the channel that is generated by each class of users is assumed to be Poisson distributed with intensities λ_f and λ_s respectively; the Poisson model is proved to be an appropriate model

for the cumulative traffic that is generated by a large population of bursty users, which is assumed to be the case in the system under consideration.

Messages are assumed to be packetized and of fixed length; it is assumed that time axis is slotted and that the beginning of a packet transmission coincides with the beginning of a slot.

All users may access the channel as long as they have a packet to transmit; the first transmission attempt takes place at the beginning of the first time slot that follows the packet generation instant. Because of the freedom that the users enjoy in accessing the channel, a transmission attempt results in either a successful packet transmission, or in a packet collision if more than one packet transmissions were attempted in the same time slot. Thus it becomes obvious that an algorithm is necessary in order for the conflicts to be resolved and the channel to remain usable.

It is assumed that all users that have a packet to transmit (and only these users need to do that) keep sensing the channel and are capable of detecting a packet collision; that is, we assume that a binary feedback information is available to all active users before the end of the current slot, revealing whether the slot was involved in a packet collision (C) or not (NC). Channel errors are not taken into consideration and packet collision is the only event that results in unsuccessful transmission.

III. Description of the Algorithm

The first time transmission policy is kept the same for both classes of users; it is simple and implies that a packet is transmitted at the beginning of the first slot following the packet generation instant. It is apparent that if the two classes are to experience different delays, they should follow different steps in the collision resolution procedure.

We are going to use a simple limited sensing collision resolution algorithm. The limited sensing characteristic is apparently important for a mobile user environment since the users may not be able to know the history of the channel before their packet generation instant. We assume that the state of a user is determined by the content of a counter that is assigned to each one of them; this counter is updated according to the steps of the algorithm and the feedback from the channel. Users whose counter content

at the beginning of a time slot is equal to one, transmit in that slot.

Let $c_i^f(c_i^s)$ denote the counter content of a high priority (regular) user, at the beginning of the i^{th} time slot. Let also $F_i, F_i \in (C, NC)$, denote the channel feedback information just before the end of the i^{th} time slot. The steps of the collision resolution algorithm consist of the following counter updating procedures that take place at the end of each time slot.

(A) If $F_i = C$ then

$$c_i^f = 1 \begin{cases} \rightarrow c_{i+1}^f = 1 & \text{with probability } \phi \\ \rightarrow c_{i+1}^f = 2 & \text{with probability } 1-\phi \end{cases}$$

$$c_i^s = 1 \begin{cases} \rightarrow c_{i+1}^s = 2 & \text{with probability } \sigma \\ \rightarrow c_{i+1}^s = 3 & \text{with probability } 1-\sigma \end{cases}$$

$$c_i^j = r \rightarrow c_{i+1}^j = r+2, \quad r \geq 2, \quad j \in (s, f)$$

(B) If $F_i = NC$ then

$$c_i^j = r \rightarrow c_{i+1}^j = r-1, \quad r \geq 1, \quad j \in (s, f)$$

The first time transmission policy can also be described by using the concept of the counter; it simply implies that a new user sets the counter equal to one at the end of the slot in which its packet arrival took place. It did not seem to us reasonable to develop different first time transmission policies for the two classes of users. It would probably be a waste of the channel capacity to give priority to rarely appearing high priority packets, before it becomes known that a collision took place. If a conflict occurs, then the collision resolution algorithm offers some priority to the high priority packets that were involved in the conflict.

From the description of the algorithm it can be easily observed that the system is of continuous entry, i.e. new users enter the system at the beginning of the first slot that follows their packet arrival, unlike what happens in the blocked access algorithms [2]; furthermore, it is obvious that the system is a last-come-first-served one. The limited sensing characteristic of the algorithm, together with the lack of need for a central controller to coordinate the users, increase the robustness and applicability of the system.

IV. Throughput Analysis

In this section we derive bounds on the stability region of the algorithm. For this purpose, we use the concept of the session and develop recursive equations to describe the operation of the system. A session is defined as a number of consecutive slots, as it is explained in the next paragraph. If μ high priority users and v regular ones attempted a packet transmission in the first slot of a session, then the pair (μ, v) determines the multiplicity of that session.

In the sequence, we define a session of multiplicity (μ, v) , $\mu \geq 0, v \geq 0$ and $\mu + v > 1$, by using the concept of a virtual stack and a marker; the stack is assumed to have infinite number of cells. We assume that the system starts operating at time $t=0$ and that the marker is placed in cell 0. The first slot involved in a packet collision marks the beginning of a session of multiplicity (μ, v) , if $\mu + v$ packets were involved in that original collision. At this time the marker is placed in either cell 3 or cell 2, depending on whether a low priority user was involved in that conflict, or not. In the sequence, the marker moves two cells upwards or one cell downwards, depending on whether the feedback was C or NC, respectively. The movement of the marker takes place at the end of a slot. The slot in which the marker moves to cell 0, is the last slot of the session. The first slot involved in a collision that will follow, marks the beginning of another session of multiplicity (μ, v) , $\mu \geq 0, v \geq 0$ and $\mu + v > 1$, if μ high priority and v low priority packets were collided. Sessions of multiplicity (μ, v) , $\mu + v \leq 1$, result in no movement of the marker and are defined to have length equal to one time slot. It should be noted that sessions cannot be identified by the users and that they are only used in the analysis of the operation of the system.

From the previous definitions it is easy to conclude that the multiplicities of the sessions are independent identically distributed random variables with probability density function $P_{\mu, v}(\mu, v) = P_{\mu}(\mu)P_v(v)$, where $P_{\mu}(F=\mu)$, $P_v(S=v)$ are Poisson density functions with parameters λ_p and λ_r respectively ; λ_p and λ_r are the cumulative input traffic rates generated by users with or without priority respectively. At this point we give the following definition for the stability region of the system.

Definition:

If for an input traffic pair (λ_r, λ_s) , the expected value of the session length of multiplicity (μ, ν) is finite, for μ and ν finite, then we say that the operation of the system is stable and the pair (λ_r, λ_s) belongs to the stability region of the system. The maximum overall sets of stable points (λ_r, λ_s) determines the maximum stable throughput region and is denoted by S_{\max} .

Let $\tau(\mu, \nu)$ denote the length of a session of multiplicity (μ, ν) . From the description of the algorithm we derive the following recursive equations.

$$\begin{aligned} \tau_{0,0} &= \tau_{0,1} = \tau_{1,0} = 1 \\ \tau_{\mu,0} &= 1 + \tau_{\phi_1+F_1, S_1} + \tau_{\mu-\phi_1+F_2, S_2}, \quad \mu \geq 2 \\ \tau_{0,\nu} &= 1 + \tau_{F_1, S_1} + \tau_{F_2, \sigma_1+S_2} + \tau_{F_3, \nu-\sigma_1+S_3}, \quad \nu \geq 2 \\ \tau_{\mu,\nu} &= 1 + \tau_{\phi_1+F_1, S_1} + \tau_{\mu-\phi_1+F_2, \sigma_1+S_2} + \tau_{F_3, \nu-\sigma_1+S_3}, \quad \mu \geq 1, \nu \geq 1 \end{aligned} \quad (1)$$

where F_i, S_i are independent Poisson distributed random variables with parameters λ_r and λ_s respectively; ϕ_i, σ_i are independent random variables that follow the binomial distribution with parameters μ, ϕ and ν, σ respectively.

Let $L_{\mu,\nu}$ be the expected value of the length of a session of multiplicity (μ, ν) . By considering the expectations of both sides of the equations in (1), we obtain the infinite dimensional linear system of equations with respect to $L_{\mu,\nu}$

$$L_{\mu,\nu} = h_{\mu,\nu} + \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,\nu} L_{k,j}, \quad \mu \geq 0, \nu \geq 0 \quad (2)$$

where $h_{\mu,\nu} = 1$ for $\mu \geq 0, \nu \geq 0$ and $a_{k,j}^{\mu,\nu} \geq 0$, for all nonnegative μ, ν, k, j , are given in Appendix A.

Since it is not possible for the above system to be solved, we will try to find bounds on $L_{\mu,\nu}$. The existence of an upper bound on $L_{\mu,\nu}$, that is finite for μ and ν finite, will provide a lower bound on the maximum stable throughput, according to the previous definition. We found that the quantity

$$\begin{aligned} L_{0,0}^u &= L_{0,1}^u = L_{1,0}^u = 1 \\ L_{\mu,\nu}^u &= \alpha\mu + \beta\nu + \gamma, \quad 1 < \mu + \nu < \infty \end{aligned} \quad (3)$$

satisfies the inequality

$$h_{\mu,v} + \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} L_{\mu,v}^u \leq L_{\mu,v}^u \quad 1 < \mu + v < \infty \quad (4)$$

for $\alpha = \alpha(\lambda_f, \lambda_s, \phi, \sigma)$, $\beta = \beta(\lambda_f, \lambda_s, \phi, \sigma)$ and $\gamma = \gamma(\lambda_f, \lambda_s, \phi, \sigma)$ properly chosen. To find proper values for α, β and γ , we substituted (3) into (4) and three categories of infinite number of inequalities were obtained. By properly choosing γ and β to satisfy some initial inequalities and by using numerical search techniques and/or the limiting inequalities as $\mu \rightarrow \infty$ and/or $v \rightarrow \infty$, we found a proper value for α , so that all three categories of inequalities be satisfied.

It is trivial to enumerate the unknown quantities $L_{\mu,v}$ that appear in (2), by finding a mapping rule from the set $\Omega = ((\mu, v), \mu \in Z_0^+, v \in Z_0^+)$ onto the set of nonnegative integers Z_0^+ . Then, the system in (2) can be written as

$$L_i = h_i + \sum_{j=0}^{\infty} a_j^i L_j \quad (5)$$

where $i \in Z_0^+$ corresponds to a specific pair (μ, v) . The references that are given throughout the analysis are related to the solutions of systems of the form (5). Since, as we have just noticed, systems (5) and (2) are equivalent, the results that appear in the references concerning the solutions of (5) extend naturally to the solutions of (2).

From the existence of the $L_{\mu,v}^u$ and since $h_{\mu,v} \geq 0$ and $a_{k,j}^{\mu,v} \geq 0$, for all nonnegative μ, v, k, j , it is implied ([7],[1]) that the infinite dimensional linear system of equations (2) has a unique nonnegative solution that satisfies

$$0 \leq L_{\mu,v} \leq L_{\mu,v}^u = \alpha\mu + \beta v + \gamma.$$

In Fig. 1, a lower bound, S_{\max}^l , on S_{\max} is plotted; the stable region includes all points (λ_f, λ_s) for which a bound, as in (3) that satisfies (4), was possible to be obtained.

If only the high priority class is using the communication channel then the system will be able to serve larger traffic ($\lambda_f \leq 0.357$) than in the case in which only low priority users were served ($\lambda_s \leq 0.32$). This is not surprising since the procedures of the algorithm are the same for both classes, with the only significant difference that there is always a waste of the first slot after a collision among low priority

users. As a consequence, there is a reduction in the maximum stable throughput when only the class of the low priority users is served.

An upper bound, S_{\max}^u , on the maximum stable throughput can also be obtained. This bound is given by the set of all pairs (λ_l, λ_s) that provide a nonnegative solution to a truncated version of the system in (2) [8], [4]. By using a large number of equations we generally obtain a tight upper bound. The upper bound that was obtained for $\mu \leq 5$, $v \leq 10$ is plotted in Fig. 1.

V. Delay Analysis

In this section, upper and lower bounds on the average packet delay are derived. It turns out that, on the average, high priority packets experience shorter delays than the regular ones.

The existence of renewal slots, under stable operation of the system, that mark the beginning of statistically identical sessions, implies that the operation of the system can be described by a regenerative process. Under these conditions, we can draw conclusions about the limiting behavior of the system by manipulating quantities that are defined on a session [9], [10]. The application of regenerative theory procedures to the delay analysis of random access algorithms, appears in [12], [4]. The same results can be obtained by using directly the strong law of large numbers [11], [7].

Let W_l and W_s be the mean cumulative delay of all high priority or regular packets respectively, that arrive in a single session; the time interval between a packet arrival instant and the beginning of the time slot that follows the packet arrival, is not included in W_l or W_s . If D_l and D_s denote the average delay of a high priority or a regular packet respectively, then from the discussion in the previous paragraph, we have

$$0.5 + \frac{W_l^l}{\lambda_l L^u} \leq D_l \leq 0.5 + \frac{W_l^u}{\lambda_l L^l} \quad (6a)$$

$$0.5 + \frac{W_s^l}{\lambda_s L^u} \leq D_s \leq 0.5 + \frac{W_s^u}{\lambda_s L^l} \quad (6b)$$

where W_l^l , W_s^l , L^l and W_l^u , W_s^u , L^u denote lower and upper bounds on the corresponding quantities; L is

the average sessions length and L is the mean packet delay until the beginning of the first time slot that follows the packet arrival.

A lower bound on L can be obtained by solving a truncated version of the system in (2). Let $L_{\mu,v}^{M,N}$ be the solution of the finite system in which only $L_{\mu,v}$ for $\mu \leq M, v \leq N$ are considered. For $(\lambda_r, \lambda_s) \in S_{\max}^1$ it is guaranteed that $0 \leq L_{\mu,v}^{M,N} \leq L_{\mu,v}$ ([8], [7]) and thus, $L_{\mu,v}^{M,N}$ is a lower bound on $L_{\mu,v}$. By assuming $L_{\mu,v}^{M,N} = 0, \mu > M$ or $v > N$, and considering the expected value of $L_{\mu,v}^{M,N}$ with respect to (μ, v) we obtain a lower bound, L^l , on the mean session length.

The quantity that appears in (3) can serve as an upper bound on $L_{\mu,v}$. This bound is arbitrary and it is generally loose. If $\tilde{L}_{\mu,v}^{M,N}$ is the solution of the finite system of equations

$$\tilde{L}_{0,0}^{M,N} = \tilde{L}_{0,1}^{M,N} = \tilde{L}_{1,0}^{M,N} = 1 \quad (7a)$$

$$\tilde{L}_{\mu,v}^{M,N} = H_{\mu,v}^{M,N} + \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} \tilde{L}_{k,j}^{M,N} \quad 0 \leq \mu \leq M, 0 \leq v \leq N, 1 < \mu+v \quad (7b)$$

where

$$H_{\mu,v}^{M,N} = h_{\mu,v} + \sum_{k=M+1}^{\infty} \sum_{j=N+1}^{\infty} a_{k,j}^{\mu,v} L_{k,j}^u$$

and $L_{k,j}^u, k > M$ or $j > N$ are given by (3), then $\tilde{L}_{\mu,v}^{M,N}$ is an upper bound on $L_{\mu,v}$ that is generally tighter than $L_{\mu,v}^u$ [4], [8]. An upper bound on L is thus obtained by considering the expected value of $\tilde{L}_{\mu,v}^u$ with respect to (μ, v) ; where

$$\tilde{L}_{\mu,v}^u = \tilde{L}_{\mu,v}^{M,N}, \quad 0 \leq \mu \leq M, 0 \leq v \leq N \quad (8a)$$

$$\tilde{L}_{\mu,v}^u = L_{\mu,v}^u, \quad \text{otherwise.} \quad (8b)$$

In the sequence, we derive bounds on W_f and W_s . Let $\omega_{\mu,v}^f$ and $\omega_{\mu,v}^s$ be the cumulative delays of the high priority and the regular packets, respectively, that arrive during a session of multiplicity (μ, v) . It is easy to observe from the description of the algorithm that $\omega_{\mu,v}^f$ and $\omega_{\mu,v}^s$ satisfy the following recursive equations.

$$\omega_{0,0}^f = \omega_{1,1}^f = 0, \quad \omega_{1,0}^f = 1 \quad (9a)$$

$$\omega_{\mu,0}^f = \mu + \omega_{F_1+0_1,S_1}^f + (\mu-\phi_1)\tau_{F_1+0_1,S_1} + \omega_{\mu-0_1+F_2,S_2}^f, \quad \mu \geq 2 \quad (9b)$$

$$\omega_{0,v}^f = \omega_{F_1,S_1}^f + \omega_{F_2,\sigma_1+S_2}^f + \omega_{F_3,v-\sigma_1+S_3}^f, \quad v \geq 2 \quad (9c)$$

$$\omega_{\mu,v}^f = \mu + \omega_{0_1+F_1,S_1}^f + (\mu-\phi_1)\tau_{0_1+F_1,S_1} + \omega_{\mu-0_1+F_2,\sigma_1+S_2}^f + \omega_{F_3,v-\sigma_1+S_3}^f, \quad \mu \geq 1, v \geq 1 \quad (9d)$$

and

$$\omega_{0,0}^s = \omega_{1,0}^s = 0, \quad \omega_{0,1}^s = 1 \quad (9e)$$

$$\omega_{\mu,0}^s = \omega_{0_1+F_1,S_1}^s + \omega_{\mu-0_1+F_2,S_2}^s, \quad \mu \geq 2 \quad (9f)$$

$$\omega_{0,v}^s = v + \omega_{F_1,S_1}^s + v\tau_{F_1,S_1} + \omega_{F_2,\sigma_1+S_2}^s + (v-\sigma_1)\tau_{F_2,\sigma_1+S_2} + \omega_{F_3,v-\sigma_1+S_3}^s, \quad v \geq 2 \quad (9g)$$

$$\omega_{\mu,v}^s = v + \omega_{0_1+F_1,S_1}^s + v\tau_{0_1+F_1,S_1} + (v-\sigma_1)\tau_{\mu-0_1+F_2,\sigma_1+S_2} + \omega_{\mu-0_1+F_2,\sigma_1+S_2}^s + \omega_{F_3,v-\sigma_1+S_3}^s, \quad \mu \geq 1, v \geq 1 \quad (9h)$$

where all variables are as defined in (1). By considering the expectations of the above equations we obtain the following infinite dimensional linear systems of equations

$$W_{0,0}^f = W_{0,1}^f = 0, \quad W_{1,0}^f = 1 \quad (10a)$$

$$W_{\mu,v}^f = g_{\mu,v}^f + \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^f, \quad \mu+v > 1 \quad (10b)$$

$$W_{0,0}^s = W_{1,0}^s = 0, \quad W_{0,1}^s = 1 \quad (10c)$$

$$W_{\mu,v}^s = g_{\mu,v}^s + \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^s, \quad \mu+v > 1 \quad (10d)$$

where $g_{\mu,v}^f, g_{\mu,v}^s$ are given in Appendix B and $a_{k,j}^{\mu,v}$, for μ, v, k, j nonnegative, are the same as in (2) and appear in Appendix A.

By following procedures similar to those that were used to obtain bounds on L , we derive upper and lower bounds of W_f and W_s . It turned out that a universal initial upper bound on $W_{\mu,v}^f$ and $W_{\mu,v}^s$ was hard to obtain. Thus, we had to divide S_{\max}^1 into several regions and derive bounds valid for (λ_r, λ_s) in a specific region. At that point, we made the assumption that the high priority traffic is much lower than the regular one and assumed λ_r to correspond to less than 20% of the total traffic. This assumption seemed to us to be realistic and bounds were obtained for the operation region of the system

$$S_{op} = \left\{ (\lambda_f, \lambda_s) : 0 \leq \lambda_f \leq 0.065, 0 \leq \lambda_s \leq \lambda_{s,max}(\lambda_f) \right\}$$

where $\lambda_{s,max}(\lambda_f)$ can be obtained from Fig. 1. The following upper bounds on $W_{\mu,v}^f$ and $W_{\mu,v}^s$ satisfying inequalities similar to those in (4), were obtained.

$$W_{\mu,v}^{f,u} = \delta\mu^2 + \varepsilon v^2 + \theta\mu + L_{\mu,v}^u, \quad \lambda_s^*(\lambda_f) \leq \lambda_s \leq \lambda_{s,max}(\lambda_f), \quad 0 \leq \lambda_f \leq 0.065 \quad (11a)$$

$$W_{\mu,v}^{f,u} = \delta\mu^2 + \varepsilon v^2 + L_{\mu,v}^u, \quad 0 \leq \lambda_s \leq \lambda_s^*(\lambda_f), \quad 0 \leq \lambda_f \leq 0.065 \quad (11b)$$

$$W_{\mu,v}^{s,u} = \delta\mu^2 + \varepsilon v^2 + \theta v + L_{\mu,v}^u, \quad \lambda_s^{**}(\lambda_f) \leq \lambda_s \leq \lambda_{s,max}(\lambda_f), \quad 0 \leq \lambda_f \leq 0.065 \quad (11c)$$

$$W_{\mu,v}^{s,u} = \delta\mu^2 + \varepsilon v^2 + L_{\mu,v}^u, \quad 0 \leq \lambda_s \leq \lambda_s^{**}(\lambda_f), \quad 0 \leq \lambda_f \leq 0.065 \quad (11d)$$

where $\lambda_s^*(\lambda_f)$ and $\lambda_s^{**}(\lambda_f)$ are some values of λ_s less than $\lambda_{s,max}(\lambda_f)$ and $L_{\mu,v}^u$ is given by (3).

The existence of the above upper bounds guarantees that the finite dimensional linear systems of equations

$$W_{\mu,v}^{f,M,N} = g_{\mu,v}^f + \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} W_{k,j}^{f,M,N}, \quad 0 \leq \mu \leq M, \quad 0 \leq v \leq N \quad (12a)$$

$$W_{\mu,v}^{s,M,N} = g_{\mu,v}^s + \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} W_{k,j}^{s,M,N}, \quad 0 \leq \mu \leq M, \quad 0 \leq v \leq N \quad (12b)$$

have a unique nonnegative solution that is a lower bound on $W_{\mu,v}^f$ and $W_{\mu,v}^s$ respectively, for $0 \leq \mu \leq M, 0 \leq v \leq N$. By using zero as a lower bound on $W_{\mu,v}^f$ and $W_{\mu,v}^s$ for $\mu > M$ or $v > N$, and considering the expectations with respect to (μ, v) , we obtain a lower bound on W_f and W_s respectively.

A tighter upper bound on W_f and W_s can be obtained, as explained earlier in this section, by solving the finite dimensional linear systems of equations

$$\bar{W}_{0,0}^{f,M,N} = \bar{W}_{0,1}^{f,M,N} = 0, \quad \bar{W}_{1,0}^{f,M,N} = 1 \quad (13a)$$

$$\bar{W}_{\mu,v}^{f,M,N} = G_{\mu,v}^f + \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} \bar{W}_{k,j}^{f,M,N}, \quad 0 \leq \mu \leq M, \quad 0 \leq v \leq N, \quad \mu+v > 1 \quad (13b)$$

and

$$\bar{W}_{0,0}^{s,M,N} = \bar{W}_{1,0}^{s,M,N} = 0, \quad \bar{W}_{0,1}^{s,M,N} = 1 \quad (13c)$$

$$\bar{W}_{\mu,v}^{s,M,N} = G_{\mu,v}^s + \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} \bar{W}_{k,j}^{s,M,N}, \quad 0 \leq \mu \leq M, \quad 0 \leq v \leq N, \quad \mu+v > 1 \quad (13d)$$

where $G_{\mu,v}^f$ and $G_{\mu,v}^s$ are given in Appendix C. An upper bound on W_f and W_s is then obtained by considering the expected value of $\bar{W}_{\mu,v}^{f,u}$ and $\bar{W}_{\mu,v}^{s,u}$ with respect to (μ,v) , where

$$\begin{aligned} \bar{W}_{\mu,v}^{f,u} &= \bar{W}_{\mu,v}^{f,M,N}, \quad 0 \leq \mu \leq M, \quad 0 \leq v \leq N \\ \bar{W}_{\mu,v}^{f,u} &= W_{\mu,v}^{f,u}, \quad \text{otherwise} \end{aligned}$$

and

$$\begin{aligned} \bar{W}_{\mu,v}^{s,u} &= \bar{W}_{\mu,v}^{s,M,N}, \quad 0 \leq \mu \leq M, \quad 0 \leq v \leq N \\ \bar{W}_{\mu,v}^{s,u} &= W_{\mu,v}^{s,u}, \quad \text{otherwise} \end{aligned}$$

The bounds on L , W_f , W_s that were obtained in this section for some values of $(\lambda_f, \lambda_s) \in S_{op}$ are shown in table 1. By substituting those bounds in (6), we calculated tight upper and lower bounds on D_f and D_s ; these bounds appear also in table 1 for some values of $(\lambda_f, \lambda_s) \in S_{op}$.

VI. Results and Conclusions

The algorithm that we developed and analyzed is supposed to operate in an environment where two classes of users with different priorities are accommodated. An algorithm for a homogeneous user population that would work in a similar way and use binary feedback information and simple splitting after a collision, has been found to achieve a maximum stable throughput of $\sim .36$ [13]. The algorithm that we suggest for a non-homogeneous population achieves total throughput, at least, between $.320 - .357$ depending on the contribution of the two classes to the total input traffic.

In Fig. 2, Fig. 3 and Fig. 4, plots of the bounds on D_f and D_s versus λ_s , for $\lambda_f=0.01$, $\lambda_f=0.03$ and $\lambda_f=0.065$ respectively, are shown. These values of λ_f correspond to an input traffic coming from the high priority class equal to $\sim 3\%$, $\sim 10\%$ and $\sim 20\%$ of the total traffic that can be served by the system. From the plots it can be observed that the high priority packets experience shorter delays than the packets of the other class; the difference is essential for $\lambda_s > .5\lambda_{s,max}$. If the nominal point of operation of the system is set around $\lambda_s = .9\lambda_{s,max}$, then the average high priority packet delay is less than half the one of the other

class.

In table 1, the delay results of the suggested algorithm are compared with the delay, D^* , that the homogeneous class equivalent algorithm (as described above), induces [13]. Again we can observe that always $D_f < D^*$ and particularly $D_f < .5D^*$ around the nominal point, the latter being defined as before.

Since privileged service is offered to some users, there has to be a price that the rest of the population must pay. The first consequence is the small reduction in the total throughput, as mentioned before. The other penalty is the increased average low priority packet delay compared with the one that the homogeneous population equivalent algorithm induces. From table 1 we can see that, indeed, $D_s > D^*$, as it was expected. The increase in D_s is far from catastrophic and it is realistic to consider that it is possible for a system to tolerate these delay increases for the low priority class, especially if strict limitations exist for the high priority users.

As an example, consider the communication system described in the second paragraph of the Introduction. Assume that the input traffic of the original class at the nominal operating point is .25 packets/packet length and thus the (desired) induced average packet delay is 5.5 - 6.0 packet lengths (last column of table 1). Assume that at night, the input rate falls to 0.065 packets/packet length. At that time, a second class of users is given permission to use the channel. If the induced average packet delay of the original class has to be at most ≈ 6.00 packet lengths, then depending on the case we observe the following: (a) If the second class has the same priority as the original, then the additional input traffic rate that can be accommodated by the system is 0.185 packets/packet length. (b) If the second class has low priority, then the additional input traffic rate becomes .25 packets/packet lengths (table 1). Thus, there is an increase by $\approx 35\%$ of the additional traffic that can be accommodated, if the population of users is divided into two classees with different priorities. The increase in the average packet delay of the low priority users is rather negligible compared to a realistic waiting time until these users are given permission to access the channel.

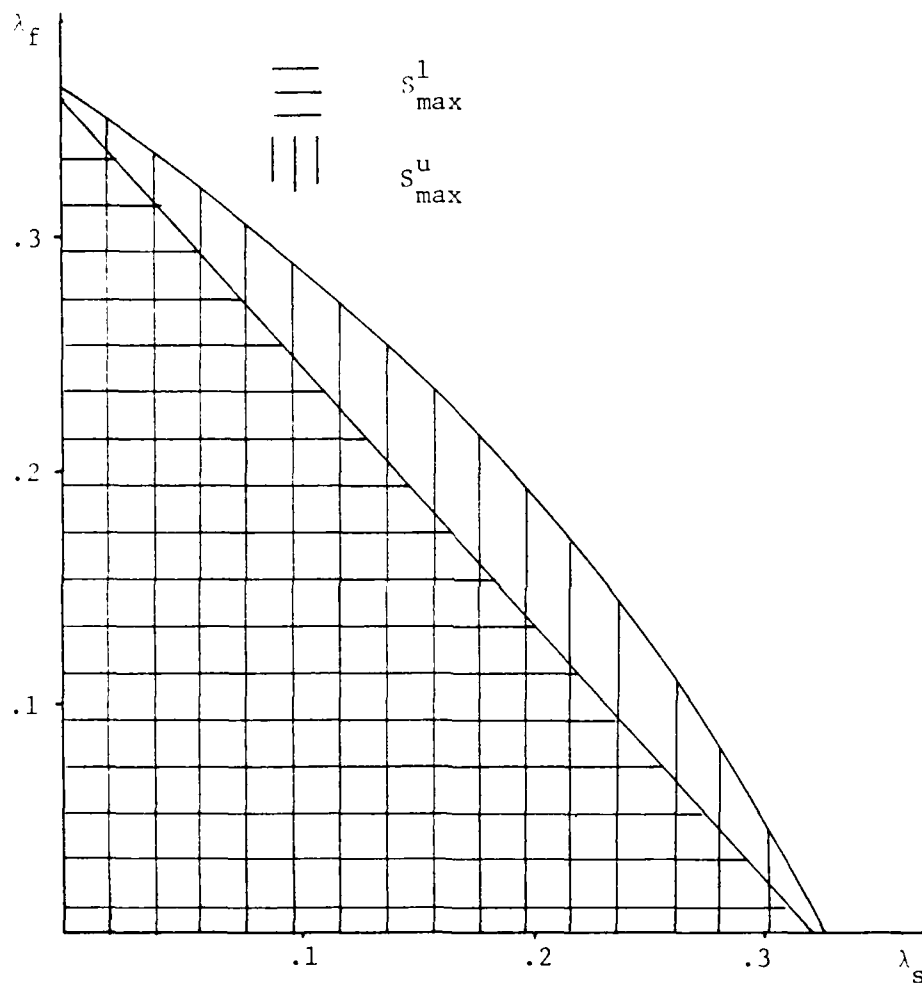


Figure 1. Upper, S_{\max}^u , and lower, S_{\max}^l , bounds on the maximum stable throughput; λ_f and λ_s are in packets/packet length.

λ_e	λ_T	λ_s	$L^I \sim L^u$	$W_f^I \sim W_f^u$	$W_s^I \sim W_s^u$	$D_f^I \sim D_f^u$	$D_s^I \sim D_s^u$	D^*
.01	.02	.01	1.000	0.010	0.010	1.555	1.590	~ 1.57
	.11	.10	1.046	0.013	0.195	1.829	2.369	~ 2.10
	.18	.17	1.176	0.019	0.663	2.186	3.815	~ 2.90
	.26	.25	1.743	0.045	4.107	3.095	9.922	~ 6.20
	.31	.30	4.386	0.232	52.435	5.793	39.793	~ 16.00
	.32	.31	7.647	0.628	185.487	8.718	78.748	~ 23.00
.03	.04	.01	1.003	0.034	0.011	1.632	1.678	~ 1.66
	.13	.10	1.063	0.046	0.220	1.951	2.571	~ 2.21
	.20	.17	1.222	0.069	0.792	2.389	4.312	~ 3.33
	.28	.25	1.987	0.189	6.052	3.672	12.681	~ 8.33
	.31	.28	3.401	0.505	27.108	5.453	28.961	~ 16.00
	.32	.29	4.845	0.961	63.802	7.113	45.905	~ 23.00
.065	.075	.01	1.013	0.085	0.013	1.800	1.878	~ 1.82
	.165	.10	1.104	0.124	0.282	2.234	3.054	~ 2.70
	.235	.17	1.338	0.208	1.159	2.900	5.595	~ 4.33
	.315	.25	2.874	0.990	16.240	5.801	23.101	~ 18.00
	.325	.26	3.719	1.619	31.505	7.200	33.080	~ 26.00

Table 1

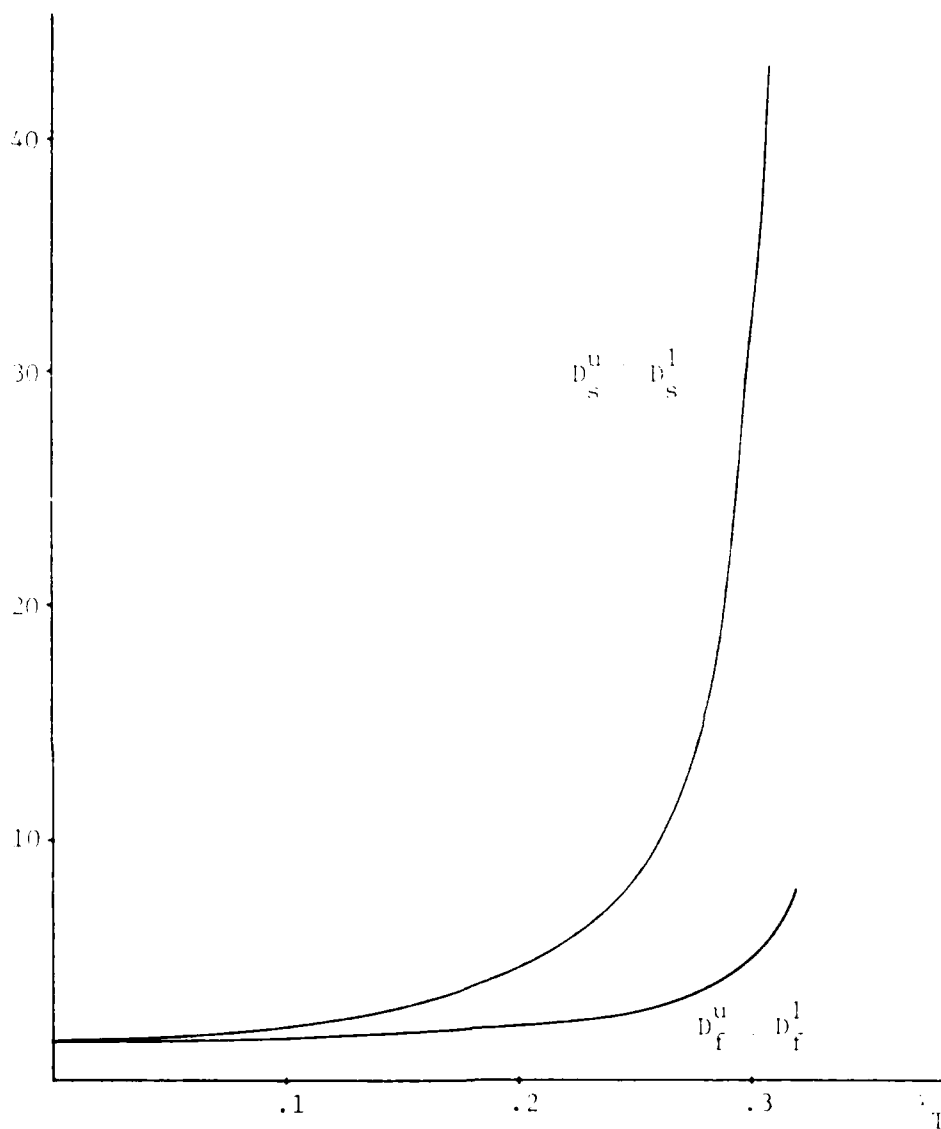


Figure 2. Average packet delay of the high, D_f , and the low, D_s , priority classes (in packet lengths), versus the total input traffic rate, λ_T , (in packet length), for $\lambda_f = .01$.

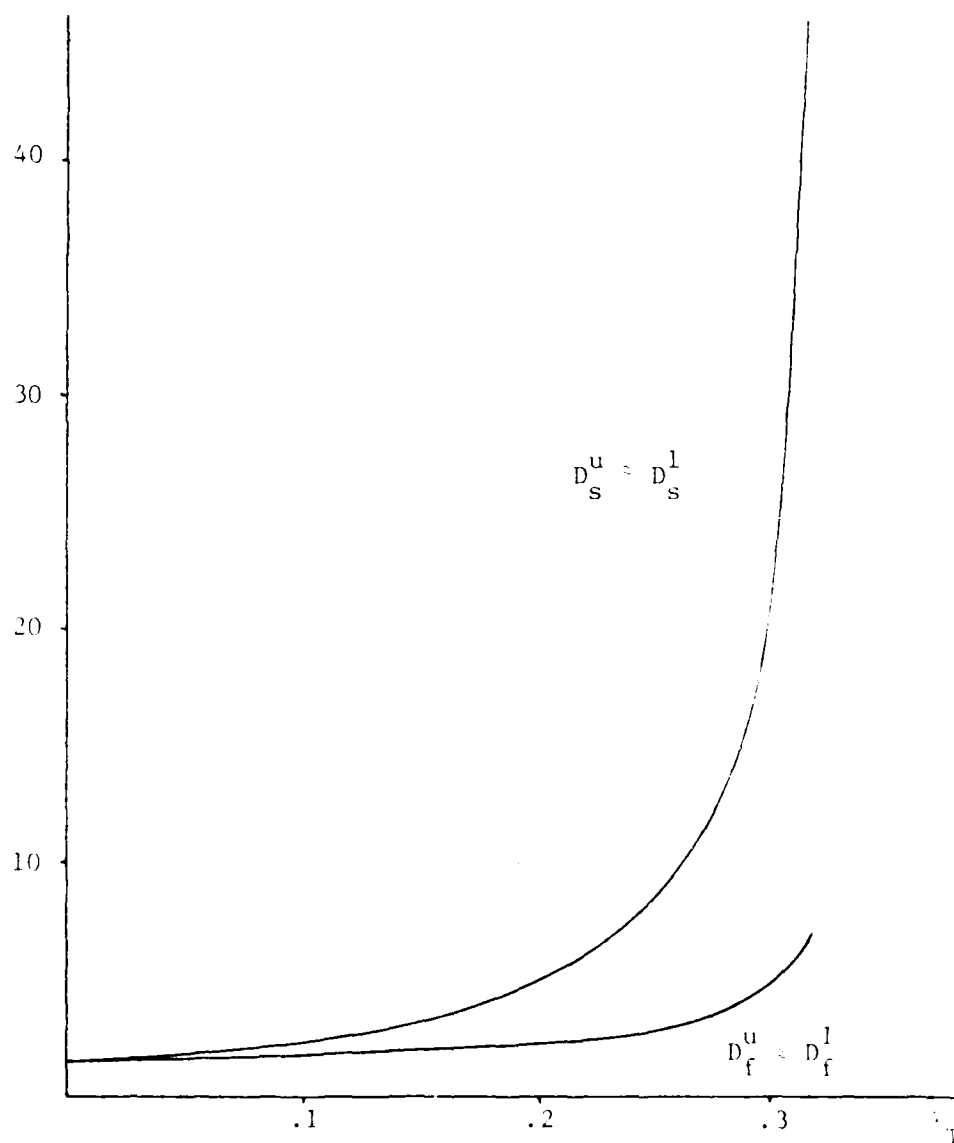


Figure 3. Average packet delay of the high, D_f , and the low, D_s , priority classes (in packet lengths), versus the total input traffic rate, λ_T , (in packets/packet length), for $\rho_f = .03$.

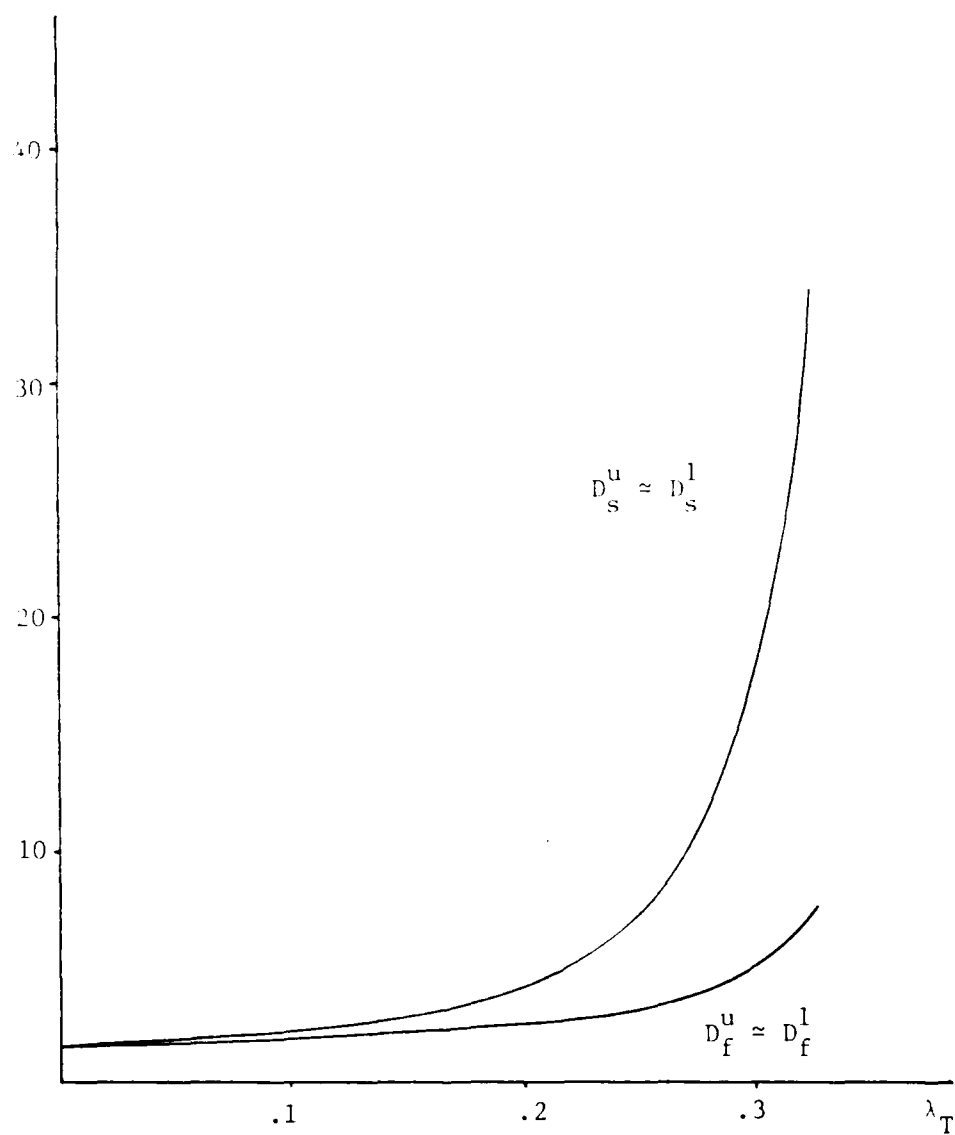


Figure 4. Average packet delay of the high, D_f , and the low, D_s , priority classes (in packet lengths) versus the total input traffic rate, λ_T , (in packets/packet length), for $\lambda_f = .065$.

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Appendix A

The coefficients of the linear system of equations that appear in (2) are given by the following expressions.

(i) For $u=0$, $v=0$ or $u=0$, $v=1$ or $u=1$, $v=0$,

$$a_{k,j}^{u,v} = 0$$

for all $k \geq 0$, $j \geq 0$.

(ii) For $u=0$, $v \geq 2$,

(i) for $0 \leq k < \infty$, $0 \leq j < \infty$

$$a_{k,j}^{u,v} = P_f(k)P_s(j) + P_f(k) \sum_{l=0}^j b_v(l)P_s(j-l) + P_f(k) \sum_{l=0}^j b_v(v-l)P_s(j-l)$$

(ii) for $0 \leq k < \infty$, $0 \leq j < \infty$

$$a_{k,j}^{u,v} = P_f(k)P_s(j) + P_f(k) \sum_{l=0}^v b_v(v-l)P_s(j-l+1) + P_f(k) \sum_{l=0}^v b_v(l)P_s(j-l+1)$$

(iii) For $u \geq 2$, $v=0$,

(i) for $0 \leq k \leq u$, $0 \leq j < \infty$

$$a_{k,j}^{u,v} = P_s(j) \sum_{i=0}^k b_u(k-i)P_f(i) + P_s(j) \sum_{i=0}^k b_u(u-k+i)P_f(i)$$

(ii) for $u \leq k < \infty$, $0 \leq j < \infty$

$$a_{k,j}^{u,v} = P_s(j) \sum_{i=0}^k b_u(u-i)P_f(k-u+i) + P_s(j) \sum_{i=0}^k b_u(i)P_f(k-u+i)$$

(c) For $k \geq 1, j \geq 1$,

(i) for $0 \leq k \leq n, 0 \leq j \leq m$

$$a_{k,j}^{(1)} = P_S(j) \sum_{i=0}^k b_{k-i} P_f(i) + \sum_{i=0}^k b_{n-k+i} P_f(i) \sum_{l=0}^j b_{j-l} P_S(l) + \\ + P_f(k) \sum_{l=0}^j b_{j-l} P_S(l)$$

(ii) for $k \leq n, 0 \leq j \leq m$

$$a_{k,j}^{(2)} = P_S(j) \sum_{i=0}^j b_{j-i} P_f(k-i+i) + \sum_{i=0}^j b_{m-j+i} P_f(k-i+i) \sum_{l=0}^j b_{j-l} P_S(l) + \\ + P_f(k) \sum_{l=0}^j b_{j-l} P_S(l)$$

(iii) for $0 \leq k \leq n, j \leq m$

$$a_{k,j}^{(3)} = P_S(j) \sum_{i=0}^k b_{k-i} P_f(i) + \sum_{i=0}^k b_{n-k+i} P_f(i) \sum_{l=0}^j b_{j-l+1} P_S(l) + \\ + P_f(k) \sum_{l=0}^j b_{j-l+1} P_S(l)$$

(iv) for $k \leq n, j \leq m$

$$a_{k,j}^{(4)} = P_S(j) \sum_{i=0}^j b_{j-i} P_f(k-i+i) + \sum_{i=0}^j b_{m-j+i} P_f(k-i+i) \sum_{l=0}^j b_{j-l+1} P_S(l) + \\ + P_f(k) \sum_{l=0}^j b_{j-l+1} P_S(l)$$

where,

$$P_f(k) = \frac{e^{-f} f^k}{k!}, \quad P_s(j) = \frac{e^{-s} s^j}{j!}$$

$$b_{i-1}(i) = \frac{i!}{i!(i-1)!} = i(1-i)^{i-1}, \quad b_{i-1}(1) = \frac{i!}{1!(i-1)!} = i^{i-1}(1-i)^{i-1}.$$

The following summations involving the coefficients $a_{k,j}^{i,j}$ were of wide use in the analysis of the algorithm.

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} = 3 \quad (i \geq 1, j \geq 1), \quad \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} = 2 \quad (i \geq 2)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{0,j} = 3 \quad (i \geq 2), \quad \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} k = 3s_f + i \quad (i \geq 1, j \geq 1)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} j = 3s_s + i \quad (i \geq 1, j \geq 1), \quad \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} k^2 = 2s_f + i \quad (i \geq 2)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} j^2 = 2s_s \quad (i \geq 2), \quad \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{0,j} k = 3s_f \quad (i \geq 2)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{0,j} j = 3s_s + i \quad (i \geq 2), \quad \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} j^2 = 2(s_s + s_s^2) \quad (i \geq 2)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{i,j} k^2 = 2(s_f + s_f^2) + i^2(1 - 2i(1-i)) + i(2s_f + 2i(1-i)) \quad (i \geq 2)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{0,j} k^2 = 3(s_f + s_f^2) \quad (i \geq 2)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{0,j} j^2 = 3(s_s + s_s^2) + i^2(1 - 2i(1-i)) + i(2s_s + 2i(1-i))$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} k^2 = 3(\lambda_f + \lambda_f^2) + \mu^2(1 - 2\mu(1-\mu)) + \mu(2\lambda_f + 2\mu(1-\mu)) \quad (\mu \geq 1, \nu \geq 1)$$

$$\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} j^2 = 3(\lambda_s + \lambda_s^2) + \nu^2(1 - 2\nu(1-\nu)) + \nu(2\lambda_s + 2\nu(1-\nu)) \quad (\mu \geq 1, \nu \geq 1)$$

Appendix B

By considering the expectation in (9), we obtain the infinite dimensional system in (10). The constants $g_{\mu,\nu}^f$ and $g_{\mu,\nu}^s$ are given by

$$g_{1,0}^f = 1$$

$$g_{\mu,\nu}^f = 0 \quad , \quad \mu=0 \quad , \quad \nu \geq 0$$

$$g_{\mu,\nu}^f = \mu + E\{(u-\tau_1)\tau_{\phi_1+F_1,S_1}\} \quad , \quad \mu > 1, \quad \nu \geq 0 \quad (B_1)$$

and

$$g_{0,1}^s = 1$$

$$g_{\mu,\nu}^s = 0 \quad , \quad \mu \geq 0 \quad , \quad \nu=0$$

$$g_{\mu,\nu}^s = \nu + E\{\nu\tau_{F_1,S_1} + (\nu-\sigma_1)\tau_{F_2,\sigma_1+S_2}\} \quad , \quad \mu=0 \quad , \quad \nu > 1 \quad (B_2)$$

$$g_{\mu,\nu}^s = \nu + E\{\nu\tau_{\phi_1+F_1,S_1} + (\nu-\sigma_1)\tau_{\mu-\phi_1+F_2,\sigma_1+S_2}\} \quad , \quad \mu > 0 \quad , \quad \nu > 0 \quad (B_3)$$

For the derivation of the upper bounds on $W_{n,n}^f$ and $W_{n,n}^s$ that are given by (11), we calculated the quantities $g_{n,n}^f$ and $g_{n,n}^s$ by using the upper bound (3) in the place of $L_{n,n} = E_{n,n}$. The upper bounds (11) are also upper bounds on the solutions of the system in (10), where the exact values of $L_{n,n}$ are used in the derivation of $g_{n,n}^f$ and $g_{n,n}^s$ [7], [8], [14]. By using the upper bound in (3) we found the following expressions.

$$\begin{aligned} (B_1) \quad g_{n,n}^f &= \alpha(1-\alpha)L^2 + [1 + \alpha + (1-\alpha)(\alpha_f + \alpha_s + \alpha) - \alpha - \\ &\quad - \alpha(b_{-}(1)P_f(0)P_s(0) + b_{-}(0)P_f(1)P_s(0)) - \alpha b_{-}(0)P_f(0)P_s(1) - \\ &\quad - \alpha(1-\alpha)]L - (\alpha+1)b_{-}(1)P_f(0)P_s(0) \end{aligned}$$

$$\text{where } L = b_{-}(0)P_f(0)P_s(0) + b_{-}(1)P_f(0)P_s(0) + b_{-}(0)P_f(1)P_s(0) + b_{-}(0)P_f(0)P_s(1)$$

$$\begin{aligned} (B_2) \quad g_{n,n}^s &= \alpha(1-\alpha)L^2 + \{(2-\alpha)(\alpha_f + \alpha_s + \alpha) - \alpha(1-\alpha) + \\ &\quad + (1-\alpha)(b_{-}(0)P_f(0)P_s(0) + P_f(0)P_s(0)) + (1-\alpha-\alpha)(b_{-}(1)P_f(0)P_s(0) + \\ &\quad + b_{-}(0)P_f(0)P_s(1) + P_f(0)P_s(1)) + (1-\alpha-\alpha)(b_{-}(0)P_f(1)P_s(0) + \\ &\quad + P_f(1)P_s(0)) + 1\}L + (\alpha+1)b_{-}(1)P_f(0)P_s(0) \end{aligned}$$

$$\begin{aligned} (B_3) \quad g_{n,n}^s &= \alpha\alpha(1-\alpha)L^2 + \alpha(\alpha+(1-\alpha)(1-\alpha))L + \{(2-\alpha)(\alpha_f + \alpha_s + \alpha) - \\ &\quad - \alpha(1-\alpha) + (1-\alpha)(b_{-}(0)P_s(0)b_{-}(1)P_f(0) + b_{-}(0)P_f(0)P_s(0)) + \\ &\quad + (1-\alpha-\alpha)(b_{-}(0)P_s(0)[P_f(1)b_{-}(1) + P_f(0)b_{-}(1-1)] + b_{-}(0)P_f(1)P_s(0) + \end{aligned}$$

$$\begin{aligned}
& + b_u(1)P_f(0)P_s(0)) + (1-\beta-\gamma)(b_v(1)P_s(0)b_u(\mu)P_f(0) + \\
& + b_u(0)P_f(0)P_s(1) + b_v(0)P_s(1)b_u(\mu)P_f(0)) + 1\}v + \\
& + b_u(1)P_s(0)b_u(\mu)P_f(0)(\beta+\gamma-1)
\end{aligned}$$

For the derivation of the lower bound on $W_{u,v}^f$ and $W_{u,v}^s$ that is given by the solution of (12) we simply substituted in (B_1) , (B_2) and (B_3) , the lower bounds on $L_{u,v}$.

Appendix C

The constants $G_{u,v}^f$ and $G_{u,v}^s$ of the systems in (13) are given by

$$G_{1,0}^f = 1$$

$$G_{L,v}^f = \sum_{k=M+1}^{\infty} \sum_{j=N+1}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^{f,u}, \quad \mu=0, \quad v \geq 0,$$

$$G_{u,v}^f = \sum_{k=M+1}^{\infty} \sum_{j=N+1}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^{f,u} + \mu + E\{(\mu-\phi_1)\tau_{\phi_1+F_1, S_1}\}, \quad \mu > 1, \quad v \geq 0$$

and

$$G_{0,1}^s = 1$$

$$G_{u,v}^s = \sum_{k=M+1}^{\infty} \sum_{j=N+1}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^{s,u}, \quad \mu \geq 0, \quad v=0$$

$$G_{u,v}^s = \sum_{k=M+1}^{\infty} \sum_{j=N+1}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^{s,u} + v + E\{v\tau_{F_1, S_1} + (\mu-\phi_1)\tau_{F_2, \sigma_1+S_2}\}, \quad \mu=0, \quad v > 1$$

$$G_{L,v}^s = \sum_{k=M+1}^{\infty} \sum_{j=N+1}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^{s,u} + v + E\{v\tau_{\phi_1+F_1, S_1} + (\mu-\phi_1)\tau_{\mu-\phi_1+F_2, \sigma_1+S_2}\},$$

$$\mu > 0, \quad v > 0$$

By substituting the values for $w_{k,j}^{f,u}$ and $w_{k,j}^{s,u}$ from (11) and $\tilde{L}_{\mu,v}$ in the place of $E\{\tau_{\mu,v}\}$ from (8), we finally obtain

$$G_{1,0}^f = 1$$

$$G_{\mu,v}^f = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} w_{k,j}^{f,u} - \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} w_{k,j}^{f,u}, \quad \mu=0, \quad v \geq 0$$

$$G_{\mu,v}^f = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} w_{k,j}^{f,u} - \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} w_{k,j}^{f,u} + \mu + \mu(1-\phi)[\alpha\phi(\mu-1) + \alpha\lambda_f + \beta\lambda_s + \gamma] - \sum_{\phi_1+F_1 \leq M} \sum_{S_1 \leq M} P_f(F_1)P_s(S_1)b_{\mu}(\phi_1)[\alpha(\phi_1+F_1)+\beta S_1+\gamma-\tilde{L}_{\phi_1+F_1,S_1}^u](\mu-\phi_1) \\ \mu > 0, \quad v > 0$$

and

$$G_{0,1}^s = 1$$

$$G_{\mu,v}^s = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} w_{k,j}^{s,u} - \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} w_{k,j}^{s,u}, \quad \mu \geq 0, \quad v=0$$

$$G_{\mu,v}^s = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} w_{k,j}^{s,u} - \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} w_{k,j}^{s,u} + v + v[\alpha\lambda_f + \beta\lambda_s + \gamma] + v(1-\sigma)[\alpha\lambda_f + \beta\sigma(v-1) + \beta\lambda_s + \gamma] + \sum_{F_1 \leq M} \sum_{S_1 \leq N} P_f(F_1)P_s(S_1)[\tilde{L}_{F_1,S_1}^u - \alpha F_1 - \beta S_1 - \gamma] + \sum_{F_2 \leq M} \sum_{\sigma_1+S_2 \leq N} P_f(F_2)P_s(S_2)b_{\mu}(\sigma_1)(v-\sigma_1)[\tilde{L}_{F_2,\sigma_1+S_2}^u - \alpha F_2 - \beta(\sigma_1+S_2) - \gamma], \quad \mu=0, \quad v \geq 1$$

$$G_{L,v}^S = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_{k,j}^{\mu,v} W_{k,j}^{S,u} - \sum_{k=0}^M \sum_{j=0}^N a_{k,j}^{\mu,v} W_{k,j}^{S,u} + v + v[\alpha\mu + \alpha\lambda_f + \beta\lambda_s + \gamma]$$

$$+ v(1-\sigma)[\alpha\mu - \alpha\mu_f + \alpha\lambda_f + \beta\sigma(v-1) + \beta\lambda_s + \gamma] + \sum_{\phi_1 + F_1 \leq M} \sum_{S_1 \leq N} P_f(F_1) P_s(S_1) b_v(\phi_1)$$

$$v[\tilde{L}_{\mu - \phi_1 + F_1, S_1}^u - \alpha(\phi_1 + F_1) - \beta S_1 - \gamma] + \sum_{\mu - \phi_1 + F_2 \leq M} \sum_{\phi_1 + S_2 \leq N} P_f(F_2) P_s(S_2) b_{\mu}(\phi_1) b_v(\sigma_1)(v - \sigma_1)$$

$$[\tilde{L}_{\mu - \phi_1 + F_2, \sigma_1 + S_2}^u - \alpha(\mu - \phi_1 + F_2) - \beta(\sigma_1 + S_2) - \gamma] \quad , \quad \mu > 0 \quad , \quad v > 0$$

The calculation of the infinite summations is carried out by using the infinite summations that appear in Appendix A.

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